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G.J. Rodin and R.A. Schapery

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Dept. of Aerospace Engineering and Engineering Mechanics, The University of Texas At Austin
Austin, TX 78712-1085

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13. ABSTRACT *(Maximum 200 words)*

This final report summarizes research on deformation and fracture models for sea ice. In many cases results are compared with those for freshwater ice. Described first is work on modeling of primary creep for relatively short loading histories, studies on the applicability of linear elastic fracture mechanics to polycrystalline freshwater and sea ice, and finally a study on stress-transmission in polycrystals undergoing grain boundary sliding. Next, emphasis is on characterization of time-dependent deformation behavior of ice from short to very long-time behavior. This behavior requires the use of models that account for broad spectrum viscoelasticity. Linear viscoelasticity theory is used first in order to develop an understanding of how single-crystal creep produces broad-spectrum behavior of polycrystals through the mechanical interaction of single crystals. Guided by these results and then nonequilibrium thermodynamic principles, nonlinear viscoelastic constitutive equations are developed that make direct use of creep compliances predicted from the linear theory of polycrystals. While only theoretical work has been done on this grant, experimental data provided by others have been used to verify some of the predictions. Comparison of theory and experiment appears in some of the publications.

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CONSTITUTIVE EQUATIONS AND FRACTURE MODELS FOR SEA ICE

G.J. RODIN AND R.A. SCHAPERY

Department of Aerospace Engineering
and Engineering Mechanics
The University of Texas at Austin 78712

1. INTRODUCTION

This final report summarizes research conducted on the project entitled "Constitutive Equations and Fracture Models for Sea Ice." Studies under the direction of Professor Gregory J. Rodin are summarized in Section 2, and those under Professor Richard A. Schapery are summarized in Section 3. Included in these sections is a listing of the papers written in the course of the project. Abstracts of these papers are given in Section 4.

Two graduate students participated in the research. Khaled Ibrahim Abdel-Tawab worked with Professor Rodin, and was awarded a Doctor of Philosology degree in December 1995; the abstract of his dissertation appears at the end of Section 4. Lu Wang worked with Professor Schapery, and was awarded a Master of Science degree in December 1995. Both students co-authored some of the publications, as shown in the publication list in the subsequent sections. Section 2 covers work on modeling of primary creep for relatively short loading histories, studies on the applicability of linear elastic fracture mechanics to polycrystalline freshwater and sea ice, and finally a study on stress-transmission in polycrystals undergoing grain boundary sliding. In Section 3 emphasis is on characterization of time-dependent deformation behavior of ice from short to very long-time behavior. This behavior requires the use of models that account for broad spectrum viscoelasticity. Linear viscoelasticity theory is used first in order to develop an understanding of how single-crystal creep produces broad-spectrum behavior of polycrystals through the mechanical interaction of single crystals. Guided by these results and nonequilibrium thermodynamic principles, nonlinear viscoelastic constitutive equations are then developed that make direct use of creep compliances predicted from the linear theory of polycrystals.

While only theoretical work has been done on this grant, experimental data provided by others have been used to verify some of the predictions. Comparison of theory and experiment appears in some of the publications.

2. RESEARCH UNDER THE DIRECTION OF G.J. RODIN.

Graduate Student: K. Abdel-Tawab (Ph.D. granted Dec. 1995).

Our research proceeded along three directions. First, nonlinear constitutive modeling of primary creep in fresh-water and transversely isotropic, columnar (S2) saline ice and identification of dominant microscopic mechanisms. Second, identification of testing conditions that lead to stress intensity factor (K)-dominance in S2 ice specimens with the emphasis on inhomogeneity and creep effects. Third, micromechanical modeling of polycrystals with weak grain boundaries to account for grain boundary sliding effects at temperatures above T=-80 C.

Modeling of Primary Creep

The objective of this research was to develop three-dimensional nonlinear constitutive equations for primary creep of fresh-water and saline S2 ice that can be used in analysis of relatively short transient load histories that are commonly encountered in ice-structure interactions. Toward this objective we proposed a dislocation creep model that reflects a competition between hardening and recovery via evolution of residual stress. This model was used as the basis for comparison of primary creep in fresh-water versus saline S2 ice and we found that the hardening strength of fresh-water S2 ice is much stronger than that of saline S2 ice while the recovery strength of fresh-water S2 ice is very close to that of saline S2 ice. Based on this observation we proposed a rate-controlling mechanism of dislocation creep in S2 ice that involves both basal and non-basal slips.

This research is presented in the paper:

- Abdel-Tawab, K. and Rodin, G.J., "Analysis of Primary Creep of Fresh-water and Saline S2 Ice," submitted to J. Cold Regions Science and Tech., 1996. (See Abstract No. (1).)

Modeling of K-Dominance

The objective of this research was to develop a set of constraints on fracture specimens and loading conditions that guarantee K-dominance and thus permit one to apply Linear Elastic Fracture Mechanics (LEFM) to S2 ice. We focused our attention to creep and inhomogeneity as the principal factors that limit the use of LEFM.

The specific objective of the creep analysis was to estimate the creep zone size as a function of the loading rate and temperature. This estimate was obtained using the creep constitutive model discussed in the previous section. Once we determined the creep zone

size as a function of the loading rate and temperature we were able to determine the minimum specimen size. We found that conditions for small-scale creep are not generally difficult to satisfy, however those conditions were not satisfied in a large number of experiments reported in the literature.

The specific objective of the inhomogeneity analysis was to examine the role of grain structure on the applicability of LEFM. To this end we identified two classes of materials. The first class includes very brittle materials in which the crack tip plastic zone is much smaller than the grain size. For such materials the specimen size is immaterial since crack growth initiation is controlled by few grains close to the crack tip. Furthermore, in brittle S2 ice, where crack growth initiation is usually intragranular, the initiation is controlled solely by the grain that contains the crack tip. For this case, we performed Monte-Carlo simulation to identify a statistical relationship between the apparent and crystallographic fracture energies. The second class of materials includes semi-brittle specimens in which the crack tip plastic zone is of the same order as the grain size. In such materials inhomogeneity modulates the standard K-fields, and in order to use LEFM we determined the relationship between the modulation magnitude and specimens size. This relationship correlated well with existing experimental observations on the minimum specimen size.

This research is presented in the papers:

- Abdel-Tawab, K. and Rodin, G.J., "On the Relevance of Linear Elastic Fracture Mechanics to Ice," IAHR 92, Proceedings of the 11th International Symposium on Ice, Vol. 3, 1992, Banff, Canada. *Int. J. Fracture*, 62, 171-181, 1993. (See Abstract No. (2).)
- Abdel-Tawab, K. and Rodin, G.J., "An Interpretation of Results of the Fracture Toughness Tests on Ice," AMD Vol. 163, *Ice Mechanics*, ASME Proceedings, 49-59, 1993. (See Abstract No. (3).)
- Abdel-Tawab, K. and Rodin, G.J., "Inelastic Effects in Fracture of Columnar-Grained Ice," AMD Vol. 207, *Ice Mechanics*, ASME Proceedings, 49-64, 1995. (See Abstract No. (4).)
- Abdel-Tawab, K. and Rodin, G.J., "The Role of Grain Structure in Brittle and Semi-Brittle Fracture of Polycrystals," submitted to J. Mech. Phys. Solids, 1996. (See Abstract No. (5).)

Micromechanical Modeling

The objective of this research was to develop a basic understanding of stress transmission in two- and three-dimensional polycrystals undergoing grain boundary sliding. In ice this phenomenon is expected to be critical at temperatures above T=-80 C and it was observed both in the laboratory and in the field. Grain boundary sliding was analyzed in the past by many researchers but our work is the first one that takes into account random geometry of polycrystals.

This research is presented in the paper:

- Rodin, G.J., "Stress Transmission in Polycrystals with Frictionless Grain Boundaries," *J. Appl. Mech.*, 62, 1-6, 1995. (See Abstract No. (6).)

3. RESEARCH UNDER THE DIRECTION OF R.A. SCHAPERY

Graduate Student: L. Wang (M.S. granted Dec. 1995).

Our research was conducted first on linear viscoelastic behavior of polycrystalline ice and then on nonlinear viscoelastic behavior. In both cases the goal was to develop physically based constitutive equations for polycrystalline sea ice subjected to uniaxial and multiaxial states of time -varying stresses which are applicable over a broad time range. A significant part of this study was concerned with prediction of all components of the creep compliance tensor; these material functions of time are needed in the constitutive equations for general time-dependent loading.

Linear Viscoelastic Creep Compliances for Transversely Isotropic Columnar (S2) ice.

The relationship between linear viscoelastic behavior of single ice crystals and their polycrystals was addressed, with emphasis on transversely isotropic columnar (S2) ice, which is a common form of ice cover in large bodies of water. The c-axes of the hexagonal ice crystals are commonly randomly oriented in the horizontal plane; the basal planes of the crystals are therefore in vertical planes. The grains are essentially columns with vertical axes. This columnar, polycrystalline ice is thus a transversely isotropic material, in which the horizontal plane is the plane of isotropy. In young sea ice brine pockets (liquid inclusions) are located along platelets which form a substructure in each grain or crystal. This structure, with brine pockets concentrated on basal planes, is easily sheared along these planes. In order to isolate the effect of basal plane shearing, we assumed all viscoelastic effects arise from this mode of deformation. The associated shear modulus

C_{44} was represented as a linear viscoelastic function, but the remaining four crystal moduli were assumed constant. First, elastic solutions for S2 ice were developed using the self-consistent method, and then the results were extended to viscoelastic behavior through the so-called correspondence principle. For comparison purposes, upper and lower bounds to the elastic polycrystal moduli were also derived. For further comparison, the Young's modulus and Poisson's ration of (isotropic) granular ice were predicted using already published equations; however, relevant theoretical results for S2 ice had not been previously published. Given the moduli of the ice crystal, in which variations in only the basal shear modulus C_{44} were considered, predictions of polycrystal viscoelastic deformations were found to be in agreement with experimental results at low stresses. It was also shown that the magnitude of transient creep of the polycrystal is not limited by mechanical interactions between grains with randomly oriented c-axes. For example, the elastic polycrystal shear modulus G vanishes when C_{44} does, with the variation $G \sim C_{44}^{1/2}$ as $C_{44} \rightarrow 0$ This behavior implies that if a single, unconstrained crystal exhibits only steady-state creep, i.e. $\varepsilon \sim$ t, the creep strain of the polycrystal varies as $t^{1/2}$ at long times; this behavior agrees with Cole's creep data on saline ice. Creep of granular ice, on the other hand, is limited by mechanical interlocking of the grains unless there are additional modes of creep deformation within or between grains.

This research is presented in the paper:

- Schapery, R.A. "Viscoelastic Deformation Behavior of Ice Based on Micromechanical Models," Proc. *Ice Mechanics*, ASME-AMD Vol. 163, pp. 15-34, 1993. Submitted to ASCE Cold Regions Engineering. (See Abstract No. (7).)

Linear Viscoelastic Creep Compliances for Anisotropic Columnar (S2) Ice.

Columnar ice with crystals having predominantly horizontal c-axes is a common form of ice cover in large bodies of water, as mentioned above. If such crystals grow as the young ice drifts freely around, or in an environment where sea currents are highly variable, the columnar crystals grow without any preferred direction in the horizontal plane. The ice is then transversely isotropic in the horizontal plane. However, in land fast ice conditions, where ice is stable relative to the sea bed and where uniform directional sea currents occur, preferred (though far from perfectly aligned) orientations of columnar crystals develop. So, in general, columnar ice is a monoclinic material with the horizontal plane as its plane of symmetry.

Here we are interested in predicting the linear elastic and viscoelastic behavior of anisotropic columnar polycrystals. Previously, Schapery made similar predictions for

transversely isotropic S2 ice. Prediction of elastic properties of anisotropic columnar ice were accomplished by the self-consistent method and a bounding method. It was shown that the Reuss and Voigt bounds are not close enough to be useful when the basal plane shear modulus c_{44} of the single crystal is small compared to its value for elastic, freshwater ice. Starting with elastic solutions and creep properties assigned only to shearing along the basal plane for each individual crystal, the uniaxial and shear creep compliances were found. They may be used in the mathematical model for polycrystals to predict the strain response to arbitrary applied stress histories. A computer program was developed which, with only a limited amount of laboratory or field data on columnar ice, permitted us to predict all thirteen elastic and viscoelastic compliances for monoclinic ice, as well as the average stresses and strains in each crystal. We found that when columnar ice is under uniaxial compression in the plane perpendicular to the column axis, the magnitude of the maximum internal tensile stress in a single crystal depends on the c-axis orientation distribution; this internal tensile stress increases with decreasing values of the basal plane shear modulus, and thus the propensity for microcracking increases with creep and with brine content.

This research is present in the paper:

- Wang, L. and Schapery, R.A., "Prediction of Elastic and Viscoelastic Properties of Anisotropic Columnar Ice," ASME-AMD Vol. 207, pp. 33-47, 1995. Submitted to ASCE Cold Regions Engineering. (See Abstract No. (8).)

Nonlinear Viscoelastic Constitutive Equations for Ice

This work was concerned with thermomechanical constitutive equations for describing time- and rate-dependent behavior of polycrystalline freshwater and sea ice. The equations which were developed provide relationships between histories of the stress tensor σ strain tensor ε and temperature T. The specific forms that were developed are explicit representations of the current ε in terms of histories of σ and T. We accounted for elastic, viscous, and delayed elastic strains. Effects of microcracking and dynamic recrystallization were not explicitly taken into account.

Experimental strain data for nine creep and recovery cycles of S2 saline ice at three different uniaxial stress levels were provided by Dempsey. The first cycle of creep behavior was used to determine material functions and constants. Excellent agreement between theory and experiment for all nine cycles was found. It clearly supports the need to use a broad spectrum creep function (such as the power law t^n), compared to creep behavior based on only a few springs and dashpots. Broad spectrum behavior arises in

part from grain-to-grain interactions, as demonstrated in our 1993 publication. (See Abstract No. 7)

The three-dimensional constitutive model that was developed is based on principles of thermodynamics and thermally activated processes, and contains as special cases some existing physically-based ice models. It was observed that what is needed at this time are additional experimental results on strain response to uniaxial and multiaxial stress histories which are different from what is normally used in ice tests. Multiple-cycle creep and recovery tests for different loading durations and different stress levels are especially convenient and useful for identification of material constants and functions and for model validation. Other test histories, such as constant stress rate and multiple step-up and step-down tests, are useful for validation purposes.

This research is presented in the paper:

- Schapery, R.A., "Thermomechanical Constitutive Equations for Polycrystalline Ice," IAHR 96, Proc. 13th Int. Symposium on Ice, Beijing, pp. 86-93. Submitted to ASCE Cold Regions Engineering. (See Abstract No. (9).)

4. ABSTRACTS

(1). ANALYSIS OF PRIMARY CREEP OF FRESH-WATER AND SALINE S2 ICE

K. Abdel-Tawab and G.J. Rodin

A simple multiaxial constitutive model for primary creep of S2 fresh-water and saline ice is proposed. This model is applied to available primary creep data for fresh-water, sea, and laboratory-grown saline ice. Results of these comparisons are used to examine microscopic and macroscopic aspects of creep of S2 ice. Also the proposed model provides dimensionless representations that can be useful in design of creep tests and presentation of creep data.

(2). ON THE RELEVANCE OF LINEAR ELASTIC FRACTURE MECHANICS TO ICE

K. Abdel-Tawab and G.J. Rodin

A new criterion is proposed which allows one to estimate the minimum size for a brittle ice fracture specimen comprised of a small number of grains. According to this criterion, linear elastic fracture mechanics is a useful theory for fresh-water ice but may have limited use for saline ice.

(3). AN INTERPRETATION OF RESULTS OF THE FRACTURE TOUGHNESS TESTS ON ICE

K. Abdel-Tawab and G.J. Rodin

Two factors which can lead to irreproducible fracture test results for ice are considered. First, under brittle fracture conditions, the irreproducibility is associated with the sensitivity of crack growth initiation to the orientation of the cleavage plane of the grain containing the crack tip. Second, the irreproducibility can be a consequence of the elastic inhomogeneity of ice lab specimens comprised of small numbers of large anisotropic grains. For each case, a useful interpretation of irreproducible fracture test results is suggested.

(4). INELASTIC EFFECTS IN FRACTURE OF COLUMNAR-GRAINED ICE

K. Abdel Tawab and G.J. Rodin

Inelasticity of S2 columnar-grained fresh-water and saline ice is studied with the objective to formulate a quantitative criterion for valid fracture toughness testing. Although the proposed criterion is not difficult to satisfy, it appears that it is often violated in laboratory tests.

(5). THE ROLE OF GRAIN STRUCTURE IN BRITTLE AND SEMI-BRITTLE FRACTURE OF POLYCRYSTALS

K. Abdel-Tawab and G.J. Rodin

The role of grain structure in brittle and semi-brittle fracture of polycrystalline specimens is examined. For brittle specimens, it is demonstrated that the Griffith criterion can only be stated unambiguously on the microscopic level. Implications of this conclusion are discussed to clarify the physical meaning of brittle fracture test results and a simple approximate analysis of the microscopic energy release rate in brittle specimens is proposed. For semi-brittle specimens, the effect of the polycrystalline inhomogeneity on the applicability of the Irwin-Orowan criterion is discussed.

(6). STRESS TRANSMISSION IN POLYCRYSTALS WITH FRICTIONLESS GRAIN BOUNDARIES

G.J. Rodin

A simple method of analysis of stress transmission in polycrystals with frictionless grain boundaries is presented. This method applies to a large class of 2-D and 3-D polycrystals which can be modeled as either periodic or disordered arrays of polyhedra. Calculations are performed for the periodic arrays of rhombic dodecahedra and truncated octahedra, and for arrays generated by the Voronoy tessellation of disordered point lattices. Results of these calculations show that normal stresses transmitted by frictionless grain boundaries are significantly different from applied stresses. In particular, it is predicted that, in disordered polycrystals subjected to uniaxial compression, 45% of the grain boundaries are in tension and the maximum tensile stress is one half of the applied stress.

(7). VISCOELASTIC DEFORMATION BEHAVIOR OF ICE BASED ON MICROMECHANICAL MODELS

R.A. Schapery

The dependence of overall viscoelastic behavior of ice on certain features of the microstructure is discussed using established methods of analysis. First, the self-consistent method and a bounding method are used to relate elastic behavior of columnar (S2) and granular ice to single crystal properties. The results are then extended to viscoelastic behavior in order to relate overall creep behavior, as well as other time and rate-dependent behavior, to microstructural parameters. Agreement with experimental results is shown. Emphasis in this paper is on linear viscoelastic behavior of S2 ice due only to intragranular creep; however, grain boundary sliding and microcracking are considered briefly.

(8). PREDICTION OF ELASTIC AND VISCOELASTIC PROPERTIES OF ANISOTROPIC COLUMNAR ICE

L. Wang and R.A. Schapery

The elastic moduli of anisotropic columnar ice are predicted in terms of single-crystal moduli and an arbitrary distribution of c-axis orientations. The self-consistent method and a bounding method are used. Specific results for the effective moduli of the polycrystal, as well as for the local stress and strain fields within an arbitrarily oriented crystal, are obtained for the following c-axis distributions: randomly oriented in the horizontal plane so that the polycrystal is transversely isotropic; along $\theta/-\theta$, or $\theta/\pm\theta$, so that orthotropic behavior results. The results are then extended to viscoelastic behavior due to intragranular, basal plane creep. The creep behavior is predicted as a function of the θ angle in the $\theta/-\theta$ and $\theta/\pm\theta$ cases. Several numerical examples are used to illustrate elastic and viscoelastic behavior.

(9). THERMOMECHANICAL CONSTITUTIVE EQUATIONS FOR POLYCRYSTALLINE ICE

R.A. Schapery

Linear and nonlinear viscoelastic constitutive equations for ice are addressed. Following a brief review of linear theory, some recently developed, physically-based

nonlinear equations are discussed, considering elastic, viscous and delayed elastic strain components. It is argued that a broad time-spectrum representation is needed for an adequate characterization of the delayed elastic strain. A specific nonlinear constitutive equation is proposed and successfully applied to the strain response of S2 saline ice subjected to multiple cycles of in-plane tensile loading and unloading.

(10). CONSTITUTIVE MODELING AND FRACTURE ANALYSIS OF ICE

Ph.D. Dissertation by K.I. Abdel-Tawab

This dissertation is concerned with primary creep and fracture behavior of columnar-grained S2 ice. In the first part of the dissertation, a simple constitutive model for primary creep is proposed. This model is based on dislocation creep mechanisms that control ice deformation under moderate stresses. In particular, the model reflects an experimentally observed transition from basal glide controlled deformation, for short-term creep, to recovery controlled deformation, for long-term creep. The model compares satisfactorily with existing experimental data for fresh-water, sea, and laboratory-grown saline ice. The model is also used to study anelastic behavior of ice. In the second part of the dissertation, we establish conditions under which linear elastic fracture mechanics (LEFM) is a valid fracture theory for ice. Under low loading rates, the applicability of LEFM is hindered by large crack tip creep zones. Accordingly, we use the constitutive model developed in the first part of the dissertation to determine the loading conditions under which a given specimen is acceptable for LEFM testing. Under high loading rates, the applicability of LEFM is hindered by microstructural features rather than creep. Under those conditions, we distinguish between brittle and semi-brittle fracture. For brittle fracture, crack growth initiation is controlled by the elastic and fracture properties of the grain containing the crack tip. To address this situation we examine the relationships between the apparent fracture toughness and the properties of the grain containing the crack tip. For semi-brittle fracture, LEFM can be used only if crack growth initiation is controlled by the asymptotic stress field operating at a distance from the crack tip that significantly exceeds the grain size. This condition is necessary to regard the specimen as essentially homogeneous. Thus, we develop a new criterion to determine a minimum specimen size that provides an essentially homogeneous response. According to our studies, LEFM is a good predictive tool for fresh-water ice, but may have a rather limited use for laboratory testing of sea ice.